## Étale cohomology reading seminar

## Exercise sheet 11

**Exercise 1.** Given a constructible  $\ell$ -adic sheaf  $\mathcal{F} = (\mathcal{F}_n)_n$  on a Noetherian scheme X, show that there exists a finite stratification such that  $\mathcal{F}_n$  is locally constant on each stratum.

**Exercise 2.** Let S be a base scheme, let Z and X be smooth S-schemes and let  $i: Z \to X$  be a closed immersion. Prove that for every point z in Z there are integers  $0 \le c \le n$ , an open neighborhood X' of z in X and a cartesian square

$$Z' \xrightarrow{i} X'$$

$$\downarrow u$$

$$A_S^{n-c} \xrightarrow{g} A_S^n$$

where  $Z' = i^{-1}(X')$ , g is given (base-changing to S) by the map of rings

$$\mathbb{Z}[X_1,\dots,X_n] \to \mathbb{Z}[X_{c+1},\dots,X_n], \qquad X_i \mapsto \begin{cases} 0 & \text{if } i \leq c \\ X_i & \text{if } i > c \end{cases}$$

and *u* is étale.